NUMERICAL ANALYSIS OF SHALLOW FOUNDATIONS ON PURELY COHESIVE SOIL UNDER ECCENTRIC LOADING

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ABSTRACT

The classical theory of the bearing capacity of shallow foundations is based on the effective width approach for the case of vertical eccentric loading. This method stipulates that the area of the foundation, used in the calculation of the bearing capacity, is equal to the area of a fictive foundation on which the loading is applied at the center. This paper evaluates the performance of this approach in predicting the ultimate load of the footing. A numerical analysis is performed to estimate the undrained bearing capacity mobilized in a purely cohesive soil under a strip footing, using the finite difference code FLAC3D (Fast Lagrangian Analysis of Continua in 3 dimensions). The results of this analysis show that the effective width approach provides a good approximation of the bearing capacity for this kind of problems.

KEYWORDS: Bearing capacity, eccentric loading, strip foundation, cohesive soil, finite difference method.

1 INTRODUCTION

The bearing capacity of shallow foundations is, usually, calculated using Terzaghi's equation [1] based on Prandlt's solution [2] and the superposition principle. This equation was derived for a strip footing resting on the soil surface in conditions of symmetry in geometry and loading. But in the presence of eccentricity of loading (see Figure 1), the problem becomes more complicated because of the detachment at the interface soil-foundation. In order to resolve this problem, the area of the foundation remaining in contact with the soil needs to be found. Seeking a solution for this problem, Meyerhof [3] conducted series of laboratory model tests and proposed an empirical procedure which is the effective width method. In this method, if the load is eccentric in the direction of one of the footing dimensions, this dimension is reduced by a double of eccentricity. To take in consideration the effect of eccentricity of loading in Terzaghi's equation, the effective width method is highly recommended (e.g. Hansen [4]; Meyerhof [5]).

Equation (1) represents the formula of the bearing capacity after introducing this method, for a purely cohesive soil. For this same type of soil, Ukritchon et al.[6] used numerical upper and lower bound methods to examine the problem of a strip footing on both uniform and non-uniform soil layers. Houlsbay & Purzin [7] used plasticity theorems to predict the limit load of footings and presented the results in terms of failure envelopes. Taiebat & Carter [8] studied the bearing capacity mobilized under strip and circular footings using the finite element method in the presence of the detachment problem. Several authors treated the effect of eccentricity of loading, also, on purely frictional soil. Amongst those authors, Loukidis et al. [9] and Krabenhoft et al.[10]. In order to estimate the bearing capacity of soil, the former ones used the finite element method and the later ones used the lower bound theorem of the limit analysis method based on the finite element method. Michalowski & You [11] developed solutions for the two previous types of soil using the theorem of the upper bound of the limit analysis method. Equation (2) represents the solution developed for the case of purely cohesive soil carrying a strip rough footing. Equations (1) and (2) have the same form with different bearing capacity factors Nc.

$$\frac{V}{A} = 5.14(1 - 2\frac{e}{B})c_u \tag{1}$$

$$\frac{V}{A} = 5.331(1 - 2\frac{e}{B})c_u \tag{2}$$

$e = \frac{M}{V}$ is the eccentricity of loading, M is the moment acting on the center the footing base, V is the vertical loading, A is the foundation's area, cu is the undrained cohesion of soil.

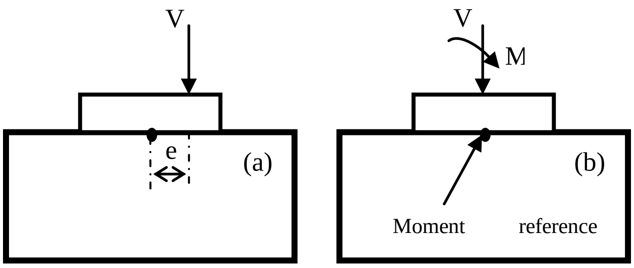

Figure1: Eccentric loading of a surface footing; (a): eccentric loading, (b): equivalent V-M loading.

In this paper, the undrained bearing capacity of strip footing resting on purely cohesive soil under eccentric loading is investigated numerically using the finite difference code FLAC3D (Fast Lagrangian Analysis of Continua in 3 dimensions) [12].

2 NUMERICAL MODEL

The choice of the code FLAC3D is justified by its efficiency in treating the problems of the bearing capacity of shallow foundations. This efficiency appears clearly in the analyses conducted by Yousssef Abdel Massih & Soubra [13], Michalowski & Dawson [14] and Puzakov et al. [15].

The geometry of the numerical model is shown in Figure 2. The footing is strip, rough, rigid and of width B resting on the soil surface. To insure that the foundation stays planar during the time of analysis, the height of the foundation is within the range of [0.2B, 0.5B] [9] and its undrained Young's modulus is 103 times the one of the soil. In order to satisfy the plane strain conditions, the foundation is considered infinitely long.

The soil is weightless, homogeneous and purely cohesive. It is modelled as an elastic perfectly plastic material obeying the Tresca failure criterion. The transmission of the load from the footing to the soil is insured by interface elements. These elements take the soil properties (cohesion) to model a rough soil-footing interface. They don't have any resistance to tension and they behave according to Coulomb shear strength criterion.

The modeling parameters are recapitulated in Table 1. Since the resulting limit load can be normalized by the foundation's width B and the soil's cohesion cu , there is no significance in the choice of their values. According to Mabrouki et al. [16], the values of the elastic properties of soil has a negligeable effect on the bearing capacity.

The height h and the width L of the discretized domain are 36B and 9B, respectively (Figure 2). It was verified that the boundaries have no effect on the limit load and the development of the failure mechanism. The boundary conditions are shown in Figure 3.a. The movement in both directions horizontal (x) and vertical (z) is not permitted for both lateral boundaries and the bottom of the model. In order to enable plane strain conditions, the displacement in the y direction is fixed for the entire model.

The numerical model is discretized into elements with different sizes. The meshing is refined in the vicinity of the foundation's edges because they are considered as singularity points. This singularity is caused by the abrupt changing in the direction of the displacement in this vicinity [9]. The mesh used in the calculations is presented in Figure 3.

The eccentric loading is applied by maintaining two vertical velocity profiles increasing linearly from zero with a defined gradient and having opposite directions (Figure 4), until plastic yielding is achieved.

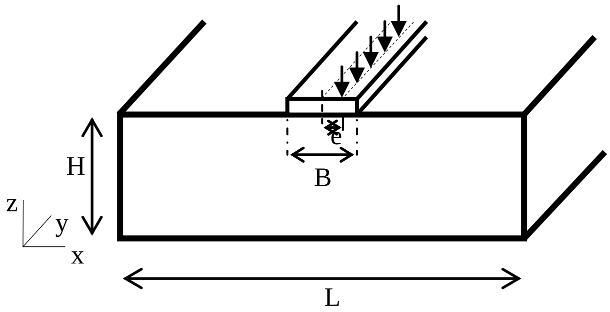

Figure2: Geometry of the numerical model.

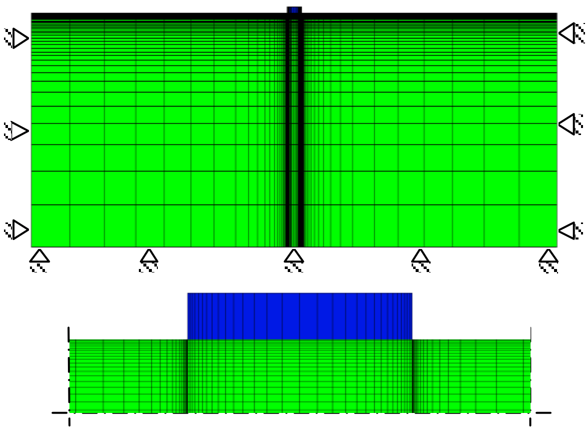

Figure3: Mesh of the numerical model ; (a) : the entire model with the boundary conditions, (b) : mesh in vicinity of the foundation.

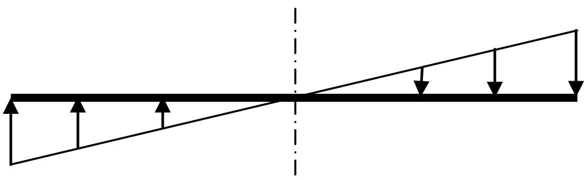

Figure4: Example of the eccentric loading application procedure (e/B =0.25).

Table1: Modeling parameters.

Parameters	Soil	Foundation	Interface
Young's modulus E (MPa)	25	25000	
Poisson's ratio υ	0.49	0.3	
Cohesion c_u (KPa)	100		100
Normal stifness k_n (Pas/m)			10^9
Tangential stifness k_s (Pas/m)			10^9

3 RESULTS AND DISCUSSION

The purpose of this analysis is to study the form of the failure envelope in loading plane (V,M), the failure mechanism and the normal stress distribution under a strip footing subjected to an eccentric loading.

The numerical bearing capacity factor Nc is equal to 5.27 overestimating the exact solution of Prandlt [2] (Nc =5.14) by 2.53%.

Figure 5 shows the numerical non-dimensional failure envelope. Every eccentricity is represented by a point with coordinates (V/Acu, M/BAcu). V and M are the vertical load and the moment acting on the center of the foundation base, respectively. They are calculated by integrating the normal stresses generated in the interface elements using a FISH function. The non-dimensional failure envelopes based on equations (1) and (2) and the one based on the results of Taibat & Carter [8] are also represented in Figure 5. All the failure envelopes are in excellent agreement for small magnitude of vertical load. However, it can be seen that for larger magnitude the effective width rule underestimates the limit load and represents a lower bound for the other methods. This same conclusion was found by Ukrichon et al.[6] and Houlsby & Purzin[7] . It can also be observed that the curve of the present study is very close to the one of Michalowski & You [11].

Figure 6 represents the normalized normal stress distribution at the soil-foundation interface with respect to the x/B ratio, with x being the coordinate (in the (x) direction) of interface points. The normalized normal stress is calculated as the ratio between the normal stress σ and the maximum normal stress σmax corresponding to each loading configuration. In case of non-eccentric loading (Figure 6.a), the stress distribution is symmetrical with respect to the centerline of the foundation with the maximum values being under the two edges of the footing. Comparing the three cases (Figure 6) allows to notice that the increase of eccentricity changes this distribution in terms of shape and size. The normal stress is reduced to zero in the vicinity of the left edge of the foundation. It is equal to zero in this area because of the absence of the transmission of loading from the footing to the soil, which can be explained by the detachement at the interface soil-foundation. With increasing eccentricity, this detachement extends over a larger area. For an ecentricity of e/B = 0.25, nearly 40% of the area of the foundation is in contact with the soil.

The evolution of the ratio V/V0 (ultimate load for different eccentricities by the ultimate load for e=0) with respect to the e/B ratio is shown in Figure 7. It appears that the effective width method is in good agreement with the numerical results. Basing on equation (1), it is clear that V/V0 is equal to (1 – 2e/B) which is a linear relationship. Equation (2), as well, gives the same relationship.

Displacement vectors represent the failure mechanisms of the previous loading cases (indicated schematically in Figure 8). In case of non-eccentric loading (see Figure 8.a), one can see the formation of a rigid elastic wedge under the base of the foundation (where the displacement vectors are vertical) and in its vicinity two fans of radial shear. It may, also, be noted that the numerical failure mechanism extend over a larger soil mass than the one mobilized by the Prandlt's failure mechanism, which might be the reason for the overestimation in the numerical bearing capacity factor Nc. Previous researchers reported the same remark and attributed it to the use of the elasto-plastic model (see Yousssef Abdel Massih & Soubra [13]). Regarding the remaining cases (Figures 8.b, 8.c), the failure mechanism is characterized by the formation of a wedge which is due to the effect of the vertical loading in addition to a scoop reflecting the moment effect. As long as the eccentricity increases, the size of the failure mechanism becomes more and more smaller, the scoop is more and more formed close to the ground surface and its pivot point of rotation is shifted towards the centerline of the footing explaining the decrease in the bearing capacity. A similar mechanism was found by Ukritchon et al.[6]. The same remark raised previously on the mobilization of a larger soil mass by the mechanism of the present study is present for the case of eccentric loading as well.

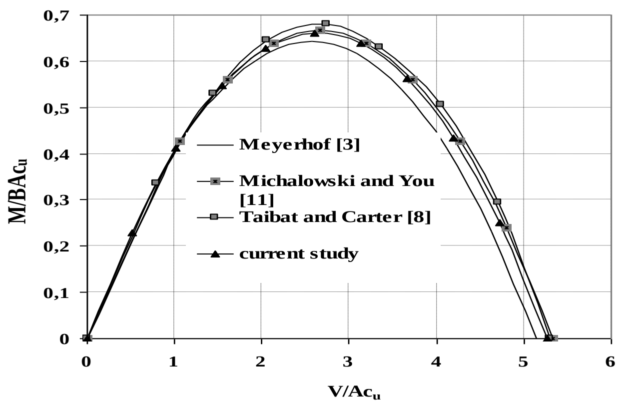

Figure 5: Non-dimensional failure envelopes in (V, M) plane.

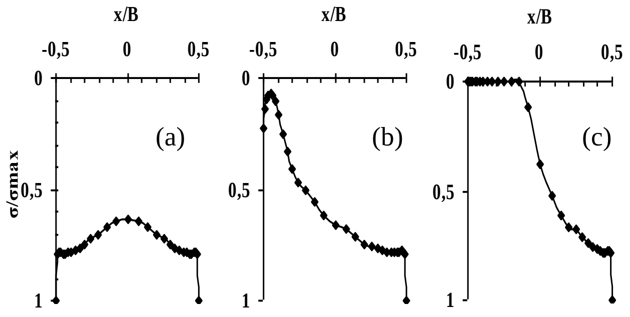

Figure6: Normalized normal stress distribution at the soil-foundation interface ; (a) : non-eccentric loading, (b) : e/B = 0.1, (c) : e/B = 0.25.

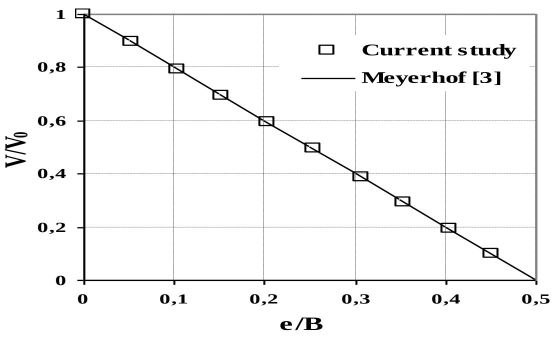

Figure7: Normalised limit load V/V_0 versus e/B ratio.

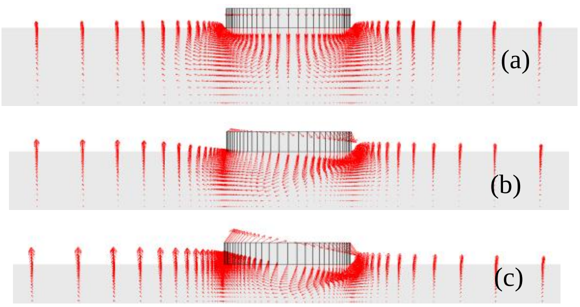

Figure8: Displacement vectors ; (a) non-eccentric loading, (b) : e/B = 0.1, (c) : e/B = 0.25.

4 CONCLUSION

A numerical simulation of the behavior of a strip footing resting on the surface of a purely cohesive soil and subjected to eccentric loading has been conducted using the finite difference code FLAC3D. This code proved to be a very powerful, efficient and precise tool for the treatment of this kind of problems. The distribution of the normal stress at the interface soil-foundation confirms that the decrease in the bearing capacity with the increase in eccentricity is due to the loss of contact pressure (detachment) between the foundation and the soil. This detachment begins at the farthest foundation's edge from the point of application of the eccentric loading and extends over a larger area with increasing eccentricity. The decrease in the numerical failure mechanism with respect to the increase in eccentricity, also, explains the decrease in the limit load. It was found that the effective width method gives a reasonable estimation for the collapse load.

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