

# VALIDATION OF THE HYBRID METHOD FOR NEAR LIGHTNING ELECTROMAGNETIC FIELD CALCULATION TAKING INTO ACCOUNT THE CONDUCTIVITY OF THE SOIL

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## ABSTRACT

In this paper we propose a hybrid method for evaluating the electromagnetic field radiated by lightning taking into account the effect of the conductivity of the soil. The proposed method is the combination of the images method and finite difference time-domain (FDTD) method. The magnetic field is obtained by the use of the images theory and the electric field is solved with using FDTD method. We calculate first, the electromagnetic field with supposing the ground as a perfect conductor and secondly we introduce the effect of the conductivity of the soil by the use of approximations formula.

**Key words:** Lightning, Electromagnetic field, FDTD method, perfect ground, finite conductivity.

## RESUME

Dans cet article nous proposons la méthode hybride pour l'évaluation du champ électromagnétique rayonné par un coup de foudre en tenant compte de la conductivité du sol. La méthode des images a pour but le calcul du champ magnétique et la FDTD pour le calcul du champ électrique. La méthode proposée est une combinaison entre la méthode des images et la méthode des différences finies dans le domaine temporel (finite difference time-domain (FDTD)). Premièrement on calcul le champ électromagnétique de la foudre en supposant que le sol est parfaitement conducteur, finalement on introduit l'effet de la conductivité finie du sol par l'utilisation des formules d'approximation.

**Mots clés :** Foudre, champ électromagnétique, méthode FDTD, sol parfait, conductivité finie.

## 1 INTRODUCTION

The calculation of lightning electromagnetic field produced by lightning is becoming more and more important in order to protect effectively the electrical and electronic systems against disturbances caused by this kind of discharges.

The computation of electromagnetic fields; radiated by lightning involves:

- Modelling the lightning return stroke which specifies the spatial-temporal distribution of the current along the lightning channel [1] [2]
- Computation of the electromagnetic field produced with supposing that the ground is infinitely conducting.

In order to taking into the count the effect of the finite conductivity of the ground we use the approximations formula which is the subject of this paper.

In this study, we will consider only the engineering models [3], [4] of lightning return stroke current essentially for two reasons. First, engineering models are characterized by a small number of adjustable parameters. Second, engineering models allows the return stroke current at any point along the lightning channel

## 2 LIGHTNING ELECTROMAGNETICFIELD COMPUTATION

### 2.1 Electromagnetic field associated to lightning

Assuming a perfectly-conducting ground, the computation of the electromagnetic fields can be greatly simplified. The components of the electric and the magnetic fields produced by a short vertical section of infinitesimal channel  $dz'$  at height  $z'$  carrying a time-varying current  $i(z', t)$ , that can be computed in the time domain using the following

relations [2]:

$$dE_r(r, \phi, z', t) = \frac{dz'}{4\pi\epsilon_0} \left[ \frac{3r(z-z')}{R^5} \int_0^t i(z', \tau - \frac{R}{c}) d\tau + \frac{3r(z-z')}{cR^4} \cdot i(z', t - \frac{R}{c}) - \frac{r(z-z')}{c^2 R^3} \cdot \frac{\partial i(z', t - \frac{R}{c})}{\partial t} \right] \quad (1)$$

$$dE_z(r, \phi, z, t) = \frac{dz'}{4\pi\epsilon_0} \left[ \frac{2(z-z')^2 - r^2}{R^5} \int_0^t i(z', \tau - \frac{R}{c}) d\tau + \frac{2(z-z')^2 - r^2}{cR^4} \cdot i(z', t - \frac{R}{c}) - \frac{r^2}{c^2 R^3} \cdot \frac{\partial i(z', t - \frac{R}{c})}{\partial t} \right] \quad (2)$$

$$dH_\phi(r, \phi, z, t) = \frac{dz'}{4\pi} \left[ \frac{r}{R^3} i(z', t - \frac{R}{c}) + \frac{r}{cR^2} \cdot \frac{\partial i(z', t - \frac{R}{c})}{\partial t} \right] \quad (3)$$

$$R = \sqrt{r^2 + (z-z')^2}$$

Where  $\epsilon_0$  and  $\mu_0$  are the permittivity and permeability of the vacuum respectively.  $c$  is the light speed.  $R$  is the distance from the dipole to the observation point, and  $r$  is the horizontal distance between the channel and the observation point.

## 2.2 Hybrid method presentation

In this paper the lightning return stroke is modelling by using engineering models.

As a first step; this method consists of evaluating the magnetic flux density at six points around the point where the electric field will be evaluated.

The magnetic field is obtained by using the images theory [5], the Simpson method is used to solve equation (3). In the second part of the method; the calculated electric field is based partially on the FDTD method [7], It's given by the following expression

$$\nabla \times \vec{B} = \mu \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = \mu \left( \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) \quad (4)$$

Where  $\mu$  is the permeability,  $\sigma$  is the conductivity,  $\epsilon$  is the permittivity,  $\vec{J}$  is the current density vector, and  $\vec{D}$  is the electric flux density vector.

The vector (4) represents a system of three scalar equations, which can be expressed in a rectangular coordinate system through their components (x, y; z) as

$$\frac{\partial E_z}{\partial t} = \frac{1}{\mu\epsilon} \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \mu\sigma E_z \right) \quad (5)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\mu\epsilon} \left( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} - \mu\sigma E_x \right) \quad (6)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\mu\epsilon} \left( \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} - \mu\sigma E_y \right) \quad (7)$$

Using the so-called central finite difference approximation [7] for space and time derivatives, and denoting  $F(m, n, p, k)$  as  $F(m\Delta z, n\Delta x, p\Delta y, k\Delta t)$ , the second-order accurate form is given by

$$\frac{\partial F(m, n, p, k)}{\partial z} = \frac{F(m+1, n, p, k) - F(m-1, n, p, k)}{\delta\ell} + O(\delta\ell)^2 \quad (8)$$

And

$$\frac{\partial F(m, n, p, k)}{\partial t} = \frac{F\left(m, n, p, k + \frac{1}{2}\right) - F\left(m-1, n, p, k - \frac{1}{2}\right)}{\Delta t} + O(\Delta t)^2 \quad (9)$$

Finally, the electric field components [7] are given by:

$$E_z\left(m, n, p, k + \frac{1}{2}\right) = E_z\left(m, n, p, k - \frac{1}{2}\right) + \frac{c^2 \Delta t}{\delta\ell} \left[ B_y(m, n+1, p, k) - B_y(m, n-1, p, k) + B_x(m, n, p-1, k) - B_x(m, n, p+1, k) \right] \quad (10)$$

$$E_x\left(m, n, p, k + \frac{1}{2}\right) = E_x\left(m, n, p, k - \frac{1}{2}\right) + \frac{c^2 \Delta t}{\delta\ell} \left[ B_x(m, n, p+1, k) - B_x(m, n, p-1, k) + B_y(m-1, n, p, k) - B_y(m+1, n, p, k) \right] \quad (11)$$

$$E_y\left(m, n, p, k + \frac{1}{2}\right) = E_y\left(m, n, p, k - \frac{1}{2}\right) + \frac{c^2 \Delta t}{\delta\ell} \left[ B_x(m+1, n, p, k) - B_x(m-1, n, p, k) + B_z(m, n-1, p, k) - B_z(m, n+1, p, k) \right] \quad (12)$$

Thus, the electric field is calculated at instant  $(k+1/2)$  taking into account the electric field at instant  $(k-1/2)$ . In order to avoid numerical instabilities, the time increment should be bounded by the grid size values. A typical choice of  $\Delta t$  is  $\Delta t \leq \Delta l / 2c$  and  $\Delta l < \lambda e$ , where  $\lambda e$  is the wavelength.

To validate this method we have using the MTLE model to modeling the lightning return stroke where there parameters [6] are listed in table 1.

**Table 1: Channel Base Current Parameters [6].**

$I_{01}$	$\tau_{11}$	$\tau_{21}$	n1	$I_{02}$	$\tau_{12}$	$\tau_{22}$	n2
(KA)	( $\mu$ s)	( $\mu$ s)		(KA)	( $\mu$ s)	( $\mu$ s)	
10.7	0.25	2.5	2	6.5	2.1	230	2

Where  $I_{01}$ ,  $I_{02}$ ,  $\tau_{11}$ ,  $\tau_{12}$ ,  $\tau_{21}$ ,  $\tau_{22}$  are constants.

Figure 1 presents the temporal variation of the current at the base of the channel.

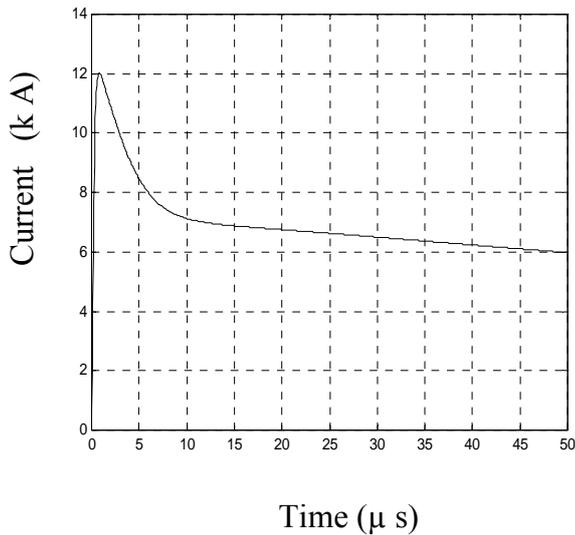
Figures 2 and 3 are the temporal and spatial variation of the channel current.

The results obtained by the hybrid method [7] using the MTL model (example shown in Table 1) are shown in Figures 4 and 5.

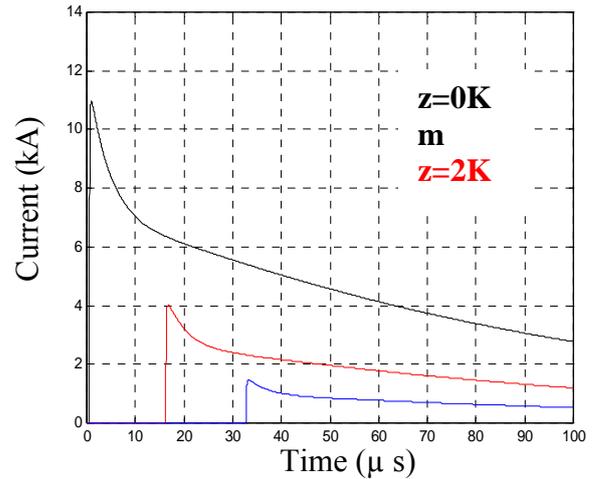
In order to compare the results obtained by our proposed method hybrid method with those obtained by the moment method in reference [7], we modled the return stroke by the TL model, the channel base current [7] is expressed by a bi-exponential equation:

$$i(0, t) = I_0 \left[ \exp(-\alpha t) - \exp(-\beta t) \right] \quad (13)$$

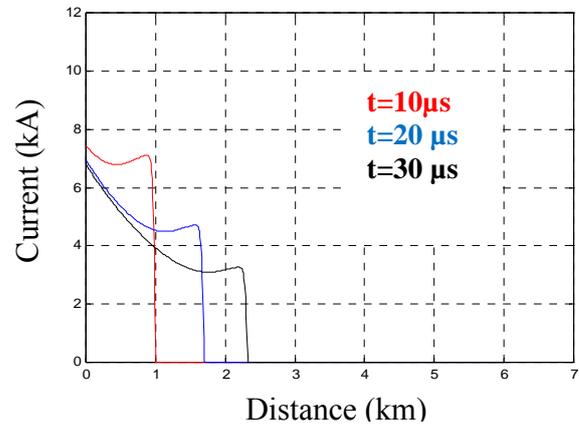
Where  $I_0 = 10$  kA;  $\alpha = 3.104$ ;  $\beta = 107$ ;  $v = 1.1 \times 10^8$  m/s.



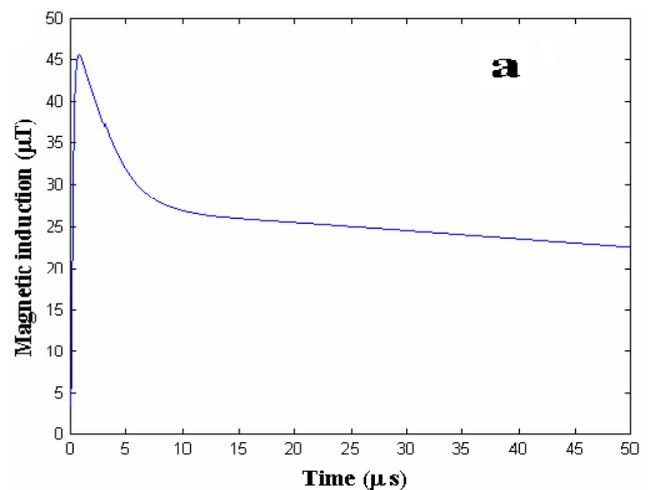
**Figure 1: Time-variation of the channel- base current.**



**Figure 2: Temporal current distribution along the lightning channel**



**Figure 3: Spatial current distribution along the lightning channel.**



**Figure 4: The magnetic field at 50 m from a lightning return stroke obtained by the proposed method [7]**

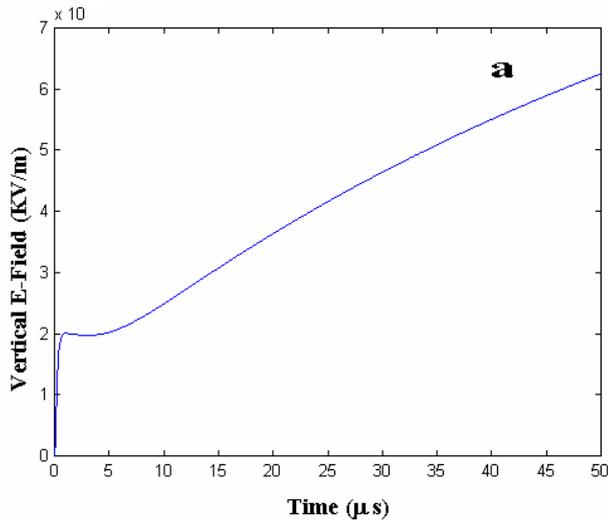


Figure 5: The electric field at 50 m from a lightning return stroke obtained by the proposed method [7].

### 3 EFFECT OF THE FINITE CONDUCTIVITY OF THE GROUND ON THE ELECTROMAGNETIC WAVE

Several study have shown that the vertical electric field component of the electric field radiated by lightning is not affected by the finite conductivity of the ground [8] but the horizontal component of the electric field is greatly affected.

In order to study the influence of the finite conductivity on the horizontal field Two approximations formula are used [9]: the Wavetilt approximation and the Rubinstein approximation.

#### 3.1 Wavetilt Approximation

The Wavetilt approximation is expressed by the equation (14):

$$W(j\omega) = \frac{E_r(j\omega)}{E_z(j\omega)} = \frac{1}{\sqrt{\epsilon_{rg} + \frac{\sigma_g}{j\omega\epsilon_0}}} \quad (14)$$

Where:

$\sigma_g, \epsilon_{rg}$  are the conductivity and the permittivity of the soil respectively.

$E_z(j\omega)$  is the Fourier transformer of the vertical electric field component for perfect ground.

$E_r(j\omega)$  is the Fourier transformer of the horizontal electric field for finite conductivity.

#### 3.1 Robinstein Approximation

The Robinstein approximation is expressed by the equation (15):

$$E_r(r, Z, j\omega) = E_{rP}(r, Z, j\omega) - H_{\phi P}(r, 0, j\omega) \frac{1 + J}{\sigma_g \cdot \delta_g} \quad (15)$$

Where:

$E_r(r, Z, j\omega)$ : horizontal electric field for finite conductivity.

$E_{rP}(r, Z, j\omega)$  horizontal electric field for perfect ground  $H_{\phi P}(r, 0, j\omega)$  azimuthal magnetic field on the soil.

$\delta_g$  : The dempth of the penetration on the soil.

The results obtained are shown in figures 6 to 10.

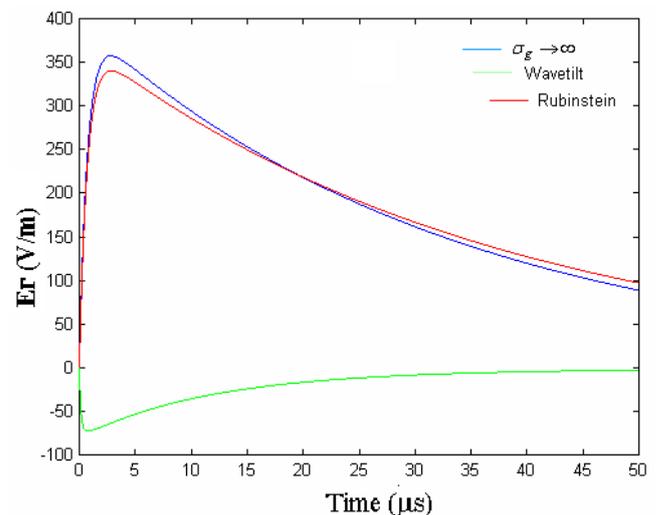


Figure 6: The horizontal electric field variation using the TL model for  $r=200m, z=10m$

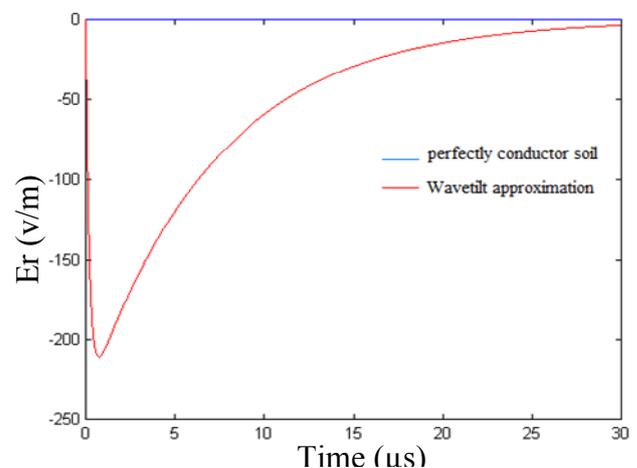


Figure 7: The horizontal electric field variation using the TL model at 100 m from lightning

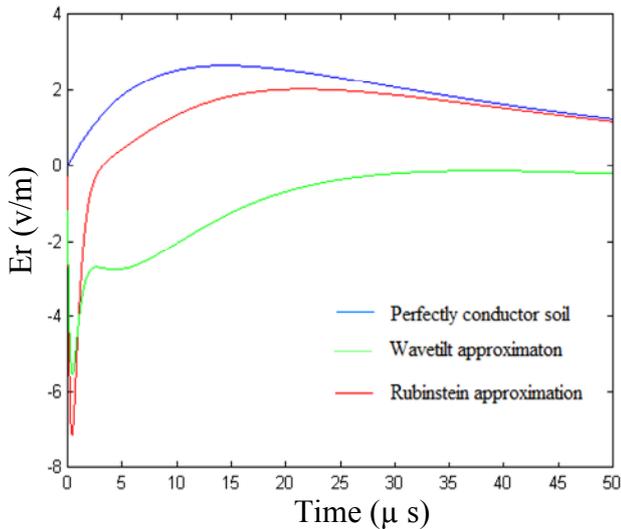


Figure 8: The horizontal electric field variation using the TL model  $r = 1.5 \text{ km}$ ,  $z = 100 \text{ m}$ .

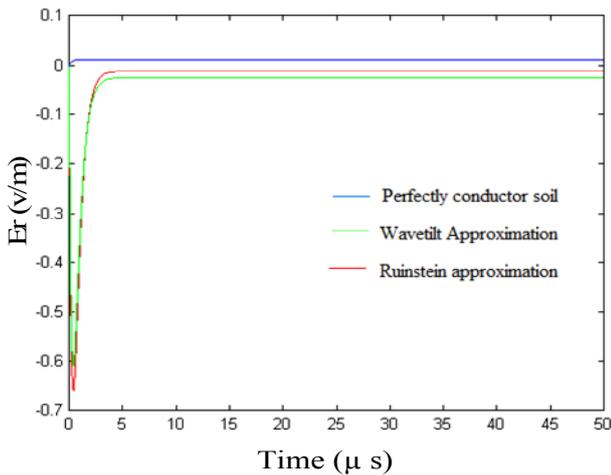


Figure 9: The horizontal electric field variation using the TL model  $r = 12 \text{ km}$ ,  $z = 6 \text{ m}$ .

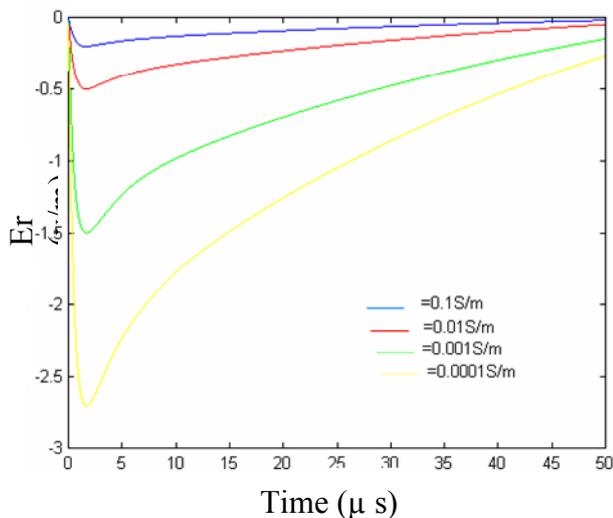


Figure 10: The horizontal electric field variation using the TL model for different conductivity with the use of Wavetilt approximation.  $r = 2 \text{ km}$ ,  $z = 6 \text{ m}$ .

#### 4 CONCLUSION

In this paper was presented initially the hybrid method which is a combination between an images theory and FDTD method. The hybrid approach makes a good alternative for the calculation of the nearest electromagnetic field radiated by the lightning channel since it allows the calculation of the electric field by the FDTD method without concern of the memory problems and the calculation precision.

It depends only on a good choice of the calculation steps. Good agreement between the compared results (cited in the references and the present work) is observed.

In order to take into account the influence of the conductor ground we have using the Wavetilt approximation and Rubinstein approximation.

From the results obtained we can conclude that:

- at a distance very close to the lightning channel and for conductivities in the order of 0.01 S/m the assumption of a perfectly ground conductor can be considered reasonable; at far distance, Wavetilt approximation can be applied to calculate the horizontal electric field with good accuracy ;
- the Rubinstein approximation can be applied to obtain satisfactory approximations of the horizontal electric field for all distances considered.

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