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Bat Algorithm for Solving IVPs of Current Expression in Series RL Circuit Constant Voltage Case

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ABSTRACT: In this paper, an efficient method for solving Initial Value Problems (IVPs) in Ordinary Differential Equations (ODEs) used in the fields of electronics and electrical engineering is demonstrated. The method is based on the Bat-Inspired Algorithm (BA), which simulates the echolocation navigation system used by bats to detect and pursue their prey. In the case of constant voltage, the IVPs arise from an RL circuit consisting of a resistor and an inductor connected in series. The suggested method's usability and effectiveness are confirmed by the experimental results obtained by numerical example. The findings reveal that the BA algorithm produces a satisfactory and precise approximation of the answers when compared to the exact solution in terms of solution quality.

Keywords: Bat Algorithm (BA), Initial Value Problems (IVP), Series RL circuit

MSC: 65-05, 65L05, 65D99.

1 Introduction

In the field of engineering science, a variety of issues are brought about by the rapid advancement of modern living and require accurate, fast solutions to challenges that are typically complicated and challenging. Therefore, optimization algorithms [19] are the means of solving this problem, with the exception of several heuristic approaches found in conventional optimization techniques, which are still insufficient. Nonetheless, because nature often finds the best solution to an issue, it is seen as a source of inspiration for tackling a variety of challenges [16].

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Because of this, several academics from various disciplines are motivated to develop a range of metaheuristic algorithms that take advantage of various operators that are modeled after natural processes [9]. An optimization issue can be solved in the simplest way possible using metaheuristic algorithms [2]. Compared to standard algorithms and iterative approaches, it can typically discover very good solutions with less computing work. It also has a high number of searched variables and a quick convergence time.

By adopting various forms in accordance with the inspired process of the systems [22], such as Particle Swarm Optimization (PSO) [13], Genetic Algorithm (GA) [7], Ant Colony Optimization (ACO) [4], Bee algorithm [12], [5], and the Flower Pollination Algorithm (FPA) [19], [20], etc.

Yang initially presented the Bat Algorithm (BA) as a substitute technique for numerical optimization in 2010 [21], [23]. It produces high echolocation, which is the technique of locating an item by reflected sound, to simulate the behavior of bats in finding prey. By producing loud noises, one may cancel out echoes that reverberate from various environments at varying frequencies [19].

To get beyond its shortcomings and capitalize on their advantages, BA was combined with other nature-inspired metaheuristic algorithms. In this regard, some adjustments have been suggested to enhance BA's performance. In [18], for instance, a BA that uses Lévy Flights and Differential Evolution (DE) operators during optimization is presented. Distribution to boost BA's search skills. A directional BA was reported in 2017 [3], suggesting the use of directed echolocation to enhance BA exploration. A noteworthy enhancement was suggested in [6], whereby the hybridization of BA and DE is employed. In order to improve the search capabilities of BA, the standard BA has also been altered to use chaotic maps rather than normal distribution [17], [14]. Additionally, a version of BA that takes the GA and Invasive Weed Optimization (IWO) [24] into account has been introduced. Numerous fields find use for BA and its expansions, including fuzzy logic, image processing, classifications, clustering and data mining, inverse problems and parameter estimation, combinatorial optimization, scheduling, and fuzzy logic in continuous optimization in engineering design.

A first order RL circuit, also known as an RL filter or RL network [1], in electronics and electrical engineering, is an electric circuit made up of an inductor and a resistor, which may be operated in parallel or series by a current source [10] or a voltage source [11]. This work is significant because it addresses the IVP that result from a series RL circuit when the voltage is constant as an optimization problem. The BA results and the results of the Range Kutta 4^{th} order approach are contrasted with the acquired results since the BA [23] is utilized as a tool to identify optimal numerical solutions to this issue.

The structure of this paper is as follows. Section 2 presents the issue formulation; Section 3 gives an overview of BA and the key procedures for estimating an IVP solution. Essential formulas and a brief explanation of series RL circuit ODEs are provided in Section 4 which exposes also an example of series RL circuit IVPs to show how BA can lead to a satisfactory result for solving IVP. The comments and conclusion are made in section 5.

2 Formulation of the problem

Let $f = f(x, y)$ be a real-valued function of two real variables defined for $a \leq x \leq b$, where a and b are finite, and for all real values of y. The equations

$$\begin{cases} y^{'} = f(x,y) \\ y(a) = y_0 \end{cases}, \tag{1}$$

are called initial-value problem (IVP); they symbolize the following problem: To find a function $y(x)$, continuous and differentiable for $x \in [a,b]$ such that $y' = f(x,y)$ from $y(a) = y_0$ for all $x \in [a,b]$ [8].

This problem possesses unique solution when: f is continuous on $[a,b] \times \mathbb{R}$, and satisfies the Lipschitz condition; it exists a real constant $k > 0$, as $\mid f(x,\theta_1) - f(x,\theta_2) \mid \leq k \mid \theta_1 - \theta_2 \mid$, for all $x \in [a,b]$ and all couple $(\theta_1, \theta_2) \in \mathbb{R} \times \mathbb{R}$.

Finding the optimal solutions numerically of an initial-value problem (IVP) is gotten with approximations: $y(x_0 + h), \ldots, y(x_0 + nh)$ where $a = x_0$ and $h = (b-a)/n$. For more precision of the solution, we must use a very small step size h that includes a larger number of steps, thus more computing time which not available in the useful numerical methods like Euler and Runge-Kutta methods [8], which may approximate solutions of (IVP) and perhaps yield useful information, often sufficing in the absence of exact, analytic solutions.

2.1 Objective function

Utilizing the finite difference formula for the derivative and equation is the algorithm's primary concept (1) we obtain,

$$\frac{y(x_j) - y(x_{j-1})}{h} \approx f(x_{j-1}, y(x_{j-1})),$$

thus,

$$\frac{y_j - y_{j-1}}{h} \approx f\Big(x_{j-1}, y_{j-1}\Big),$$

consequently, we have to consider the error formula:

$$\left[\frac{y_j - y_{j-1}}{h} - f\Big(x_{j-1}, y_{j-1})\Big)\right]^2.$$

The objective function, associated to $Y = (y_1, y_2, ..., y_d)$ will be:

$$F(Y) = \sum_{j=1}^{d} \left[\frac{y_j - y_{j-1}}{h} - f\Big(x_{j-1}, y_{j-1}\Big)\right]^2. \tag{2}$$

2.2 Consistency

We are interested in the calculation of $Y = (y_1, y_2, ..., y_d)$ which minimizes the objective function equation (2). We have from Taylor's formula order 1;

$$y_j = y_{j-1} + hy'_{j-1} + O\left(h^2\right), j = 1, \cdots d.$$

So,

$$\frac{y_j - y_{j-1}}{h} = y'_{j-1} + O(h)$$

If we subtract $f(x_{j-1}, y_{j-1})$ from both sides of last equation, we obtain

$$\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1}) = y'_{j-1} - f(x_{j-1}, y_{j-1}) + O(h), j = 1, \cdots d$$

The last relation shows that the final value $Y = (y_1, y_2, \cdots, y_d)$ is an approximate solution of IVP, for small value of h.

3 BAT ALGORITHM OVERVIEW

Yang introduced the bat method in 2010 [23]. Given that microbats are capable of producing high echolocation, it mimics their echolocation behavior. This advantageuse algorithm may be summarized as [21].

3.1 Idealized rules of BA

1) All bats sense distance via echolocation, and they also somehow magically 'know' the difference between background obstacles and food/prey.
2) Bats look for food by flying randomly at position x_i at velocity v_i, fixed frequency f_{min}, changing wavelength λ, and loudness A_0. Depending on how close their target is, they may automatically modify the wavelength (or frequency) and rate of pulse emission $r \in [0, 1]$.
3) We assume that the loudness changes from a big (positive) A_0 to a minimal constant value A_{min}, even though it might fluctuate in many other ways.

3.2 Mathematical equations

Virtual bats are moved in accordance with the following equations to generate new solutions:

$$f_i = f_{\min} + (f_{\max} - f_{\min})\,\beta$$
$$v_i^t = v_i^{t-1} + \left(x_i^t + x_*\right) f_i$$
$$x_i^t = x_i^{t-1} + v_i^t$$

Where β is a random vector selected from a uniform distribution with $\beta \in [0, 1]$. After comparing every answer among every bat, x_* is the current global best position (solution). the equation's present optimal solution

$$x_{new} = x_{old} + \partial A^t$$

where A^t is the average loudness of all the best at this time step, and $\partial \in [-1, 1]$ is a random value. The volume may be adjusted to any convenient level since, after a bat has discovered its target, it normally becomes quieter while its rate of pulse emission rises.

3.3 Pseudo code of BA

Objective function $f(x)$, $x = (x_1, ..., x_d)^T$

Initialize the bat population $x_i (i = 1, 2, ..., n)$and v_i

Define pulse frequency $f_i$at x_i

Initialize pulse rates $r_i$and the loudness A_i

while ($t < Maxnumberofiterations$)

Generate new solutions by adjusting frequency,

and updating velocities and locations/solutions

    if ($rand > r_i$)

    Select a solution among the best solutions

    Generate a local solution around the selected best solution

    end if

    Generate a new solution by flying randomly

    if ($rand < A_i$& $f(x_i) < f(x_*)$)

    Accept the new solutions

    Increase ri and reduce A

    end if

Rank the bats and find the current best x_*

end while

Postprocess results and visualization

4 NUMERICAL APPLICATION

All calculations for our experimental investigation were carried out using an Intel Duo Core 2.20 GHz PC running MSWindow 2007 Professional and the Matlab environment version R2013a compiler. Graphical and tabular representations of the numerical results are provided. The BA findings are shown in table 2 in comparison to the precise outcomes for the problem under study as well as, the absolute error. Two kinds of parameters are required for the issue treatment: the first is connected to BA , and the second is related to IVP provides. the parameter settings needed to produce the BA is presented via Table 1. Tables 3 and 4 are for computational time results and statistical analysis results respectively.

4.1 Solving Series RL Circuit ODE's as application

In Figure 1, an inductor and a resistor are linked in series to create the RL circuit. When the switch is closed, a constant voltage V is applied [1]. The voltage across the resistor is given by$V_R = Ri$. The voltage across the inductor is given by $V_L = L\frac{di}{dt}$. Kirchhoff's voltage law says that the directed sum of the voltages around a circuit must be zero. This results in the following differential equation [11]:

$$Ri + L\frac{di}{dt} = V.$$

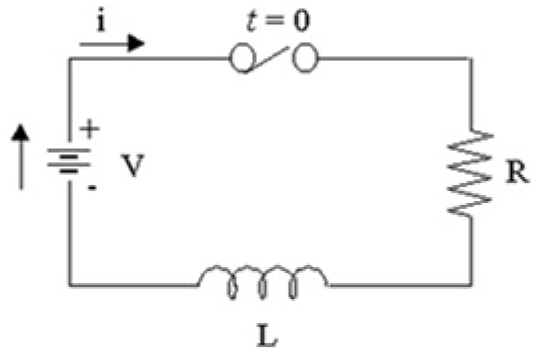


Fig. 1. The RL circuit diagram

Once the switch is closed, the current in the circuit is not constant. Instead, it will build up from zero to some steady state. The solution of this differential equation is

$$i = \frac{V}{R}(1 - e^{(-(\frac{R}{L})t)}). \tag{3}$$

Example study: A series RL circuit with $R = 50\Omega$ and $L = 10H$ has a constant voltage $V = 100V$ applied at $t = 0$ by the closing of a switch. We want to find the current expression in this case, then the formula 3 can be used here only because the voltage is constant and can not work in the alternative case. For application needs eq. 4 is given:

$$i = 2(1 - \exp(-5t)). \tag{4}$$

When plotting eq. 4, the graph shows the transition period during which the current adjusts from its initial value of zero to the final value $\frac{V}{R}$, which is the steady state [10]. The time constant (TC), known as τ of the function is the time at $\frac{R}{L}$ is unity $(= 1)$. Thus for the RL transient, the time constant is $\tau = \frac{L}{R}$ Seconds In this example, the time constant, TC, is 0.2 Seconds.

4.2 Related parameters

BA is a tool for optimization. After that, the discretization form of the fundamental differential equation is converted. When the derivative term in the discretized form is substituted by a difference quotient for approximations, the differential equation may be transformed into discretization form using the backward difference formula.

The following are the parameters linked to IVP:

1) The length of the IVP interval, $h = (b - a)/(n + 1)$, is an evenly partitioned subinterval of length $(n + 1)$.
2) There are nine internal nodes.
3) $h = 0.2$ is the step size.
4) The differential equation is solved between $t > 0$ and the initial condition, $i = 0$ for $t = 0$.

Parameter	Quantity
Dimension of the search variables (d)	10
Number of generations (N)	1000
Population size (n)	20
Loudness (constant or decreasing) (A)	0.5
Pulse rate (constant or decreasing) (r)	0.5

TABLE 1

Parameters adopted by (BA).

i	x_i	$Exact Results$	$Numerical Results$		$Absolute Error$	
			BA	$RK4$	BA	$RK4$
0	0.00	0.0000	0.0000	0.0010	0.0000	0.001
1	0.14	1.0068	1.0074	1.0079	0.0006	0.0011
2	0.28	1.5068	1.5076	1.5083	0.0008	0.0015
3	0.42	1.7551	1.7560	1.7569	0.0009	0.0018
4	0.56	1.8784	1.8794	1.8805	0.0010	0.0021
5	0.70	1.9396	1.9408	1.9421	0.0012	0.0025
6	0.84	1.9700	1.9715	1.9729	0.0015	0.0029
7	0.98	1.9851	1.9869	1.9884	0.0018	0.0033
8	1.12	1.9926	1.9946	1.9963	0.0020	0.0037
9	1.26	1.9963	1.9986	2.0004	0.0023	0.0041
10	1.40	1.9982	2.0008	2.0028	0.0026	0.0046

TABLE 2

Numerical Results of the Example for d=10

5) The role of the objective

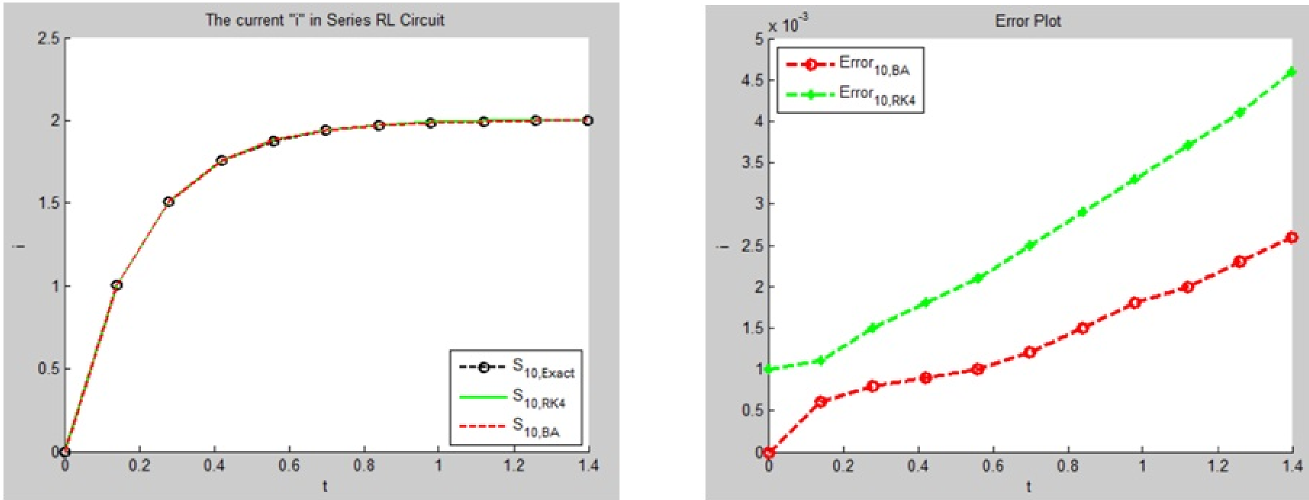
$$\begin{aligned} F\left(y_1, y_2, ..., y_{10}\right) &= \sum_{j=1}^{10}\left(\frac{y_j - y_{j-1}}{h} - f\left(x_{j-1}, y_{j-1}\right)\right)^2 \\ &= \sum_{j=1}^{10}\left(\frac{y_j - y_{j-1}}{h} - y_{j-1}\right)^2 \end{aligned}$$

The parameters adopted by BA in the treated example are summarized in Table 1:

4.3 Numerical results

The comparison between the performances of BA and $RK4$ face to the exact results are shown in the Table 2 and their graphical representations is in the Figure 2 In both representations of the results confirm that BA is better than $RK4$ because it has a very close curve to the exact curve contrary to $RK4$ method. BA method offers a very negligible absolute error compared to $RK4$ method.

The findings of our simulations show that BA is straightforward, adaptable, and simple to apply. It also saves time by rapidly achieving convergence at an early stage and transitioning from exploration to exploitation. It provides encouraging best practices for resolving IVP.



a)- Exact results, BA and RK4 results.

b)- Absolute error.

Fig. 2. Application example results.

Algorithm	Time
BA	4.687
$RK4$	5.039

TABLE 3

Average computational time of up to 50xD iterations by the used algorithms using 50 trials for the example

4.4 Computational time results

The average computing time (in seconds) for each of the 50 distinct trials of the chosen algorithms for the examined case, calculated using the dimensional space $D = 10$, is shown in Table 3. It is evident that the BA algorithm maintained a computing time that was competitive when compared to the Range Kutta 4^{th} $(RK4)$ order approach, which is a significant benefit that stems directly from straightforward population update processes.

4.5 Statistical analysis

This section of the paper deals with the statistical analysis of data acquired by the proposed BA and compared to the $RK4$ technique after demonstrating the advantage of BA with regard to computing times. These studies should give enough information to understand how BA works better than the $RK4$ technique. In the dimension $D = 10$, table 4 presents the mean and standard deviation of the difference between the computed optimum values and the genuine optimum values. The best-performing algorithm, according to the results, was BA.

Algorithm	Mean	STD
BA	0.8334	0.9570
$RK4$	0.8339	0.9573

TABLE 4

Statistical results obtained for the studied example over Dim =10.

5 Conclusion

This study discusses the usage of standard BA for solving IVPs when it's applied as a tool to numerically optimize the IVPs that arise in the field of electrical engineering and are ODEs of the series RL circuit in the voltage constant case through a selected example. When the precise answers, algorithmic results, and $RK4$ method results were compared, it was shown that BA outperformed $RK4$ method by providing exact solutions with the least amount of error.

BA excels in handling complex issues and has a remarkable capacity to tackle a wide range of problems, including highly nonlinear situations. Further research on BA will enhance the algorithm through profound studies on parameter tuning, parameter control, accelerating coverage, adding Bat smell observation property, using a wider variety of parameters, conducting more thorough comparison studies with more open-source algorithms, and so on. Additionally, BA ought to be used in a number of engineering and industrial optimization applications.

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Declaration

The author declares no conflict of interest.

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